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TRANSFORMS TO RADAR SIGNAL PROCESSING*

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DTIC QUALITY INSPECTED 3

19961024 068

# HUMAN TRANSLATION

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# Prospects in the Application of Wavelet Transforms to Radar Signal Processing*

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Abstract: Developments of signal analysis and wavelet transform from the viewpoint of time-frequency analysis are surveyed, and the superiorities of wavelet transform as applied to signal processing are investigated with a focus on the potential applications of wavelet transform to radar signal processing, especially its application to wideband or ultra-wideband radars. Further, radar ambiguity function analysis, signal detection and parameter estimation, and recognition of radar targets using wavelet transforms are discussed. Finally, the prospects of applications of wavelet transforms to signal processing are examined.

Key words: radar targets; signal detection; parameter estimations/wavelet transform; short time Fourier transform; Wigner-Ville distribution; ambiguity function; target identification

Classification Number    TN957.51

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1

An FFT can obtain a signal frequency spectrum distribution only as a whole, but cannot make a local analysis for a time varying and non-equilibrium signal. While the short-time Fourier transform (STFT) and Wigner-Ville distribution (WVD) seem to be a significant breakthrough in the time-frequency analysis of a time-varying signal, yet several superimposed signals may cause serious cross term interference [1] and severely affect their wide application. In recent years, however, researchers have focused their efforts on a totally new approach, that is the wavelet transform which can overcome the above problem. The wavelet transform, refereed to as a "Mathematical Microscope", possesses a localization character in both the time domain and frequency domain, and it also gradually adopts a fine time domain or air domain sampling step length for high-frequency components and can focus on any part of a target.

Wavelet theory can provide a unified framework for multiple resolution analysis and time-frequency analysis and is likely to have wide applications. Then what are its application prospects in radar signal processing? Modern radar technology is progressing toward the goal of ultra-wideband (high resolution), multifunction and intelligence. The echo wave signals which a radar receives, different from those from a regular point target, are synthetic images (one-dimensional, two-dimensional and three-dimensional), with multiple scattering centers, of local fine features. Thus, the performance improvement based on the conventional FFT technique is quite limited and particularly, the regular FFT can by no means analyze the gradually changing pulsed echo waves received from wideband signals, such as those from an impact radar. In this case, wavelet transforms have unique advantages in analyzing such signals.

Also, wavelet transforms can execute local feature extraction from the one-dimensional image echo wave data received by the radar, so as to realize target parameter estimation,

detection and identification. In addition, it can perform image processing, compression, coding and matching for the two-dimensional and three-dimensional images received and provide an effective means for future radar intelligent signal processing.

On the other hand, the nonlinear signal processing method based on the subform and chaos challenges the conventional radar clutter statistical processing. This theory states that the radar echo waves, coming from the clutter source of a reflecting body with subform features, also have subform features [2-5]. With in-depth study and a breakthrough, this theory will surely mean a leap in quality in radar signal processing. While wavelet transforms can serve as a means of critical mathematical analysis for the practicability of the subform and chaos theory. Therefore, combined research on the wavelet transform and radar signal processing has tremendous potential and is regarded as a completely new area of research.

1. Wavelet Transform and Radar Signal Processing

With advances in wideband and ultra-wideband radar technology, a target image with a fine (high resolution) structure (one-dimensional, two-dimensional or three-dimensional) can be derived. This high-resolution image has more local undulation features compared with the conventional point target echo waves. Particularly, the echo wave signals obtained by an impact radar or noise radar are transient and also a non-smooth process in a broad sense [6-11]. In this case, the wavelet transform technique is needed for their analysis to upgrade the performance of radar signal processing. Additionally, D. L. Jaggard et al. revealed that surface forms like natural landforms, clouds and sea surface are subform targets which can be described with the finite form of Weierstrass subform function, and that the echo wave signal time series from the interaction between those subform targets and electromagnetic

3

waves also shows subform features [2-5]. Subsequently, the wavelet transform "microscope" technology is also needed in the research on the multiple sub-dimensional anomalous indexes and distribution function scale indexes of these subform models [10].

In the first place the wideband signal must be defined. Suppose there is a signal s(t) and its corresponding Fourier transform is s($\omega$), then a definition can be derived for two bandwidths:

Noise bandwidth

$$B_n = \int_{-\infty}^{+\infty} |s(\omega)|^2 d\omega \Big/ |s(0)|^2 \qquad (1)$$

Signal bandwidth

$$B_s = \int_{-\infty}^{+\infty} \omega^2 |s(\omega)|^2 d\omega \Big/ \int_{-\infty}^{+\infty} |s(\omega)|^2 d\omega \qquad (2)$$

and then the wideband signal is defined to meet the following condition:

$$\max(B_n, B_s) >> f_0 \qquad (3)$$

where $f_0$ is the carrier wave frequency. The multiple signal that can satisfy Eq. (3) can be expressed by the product of the complex envelope signal and the modulated part of the carrier wave; otherwise, that multiple signal will not have a distinctive complex envelope.

The foregoing conclusion is obvious for a sonar signal; Eq. (3) is difficult to meet for radars below the decimeter level. Thus, general signals located in the front end of the radar receiver can be regarded as narrow-band signals. Nonetheless, if the received radar echo waves have undergone frequency mixing and back-level zero midband processing, then the output signals can be considered as wideband signals and the related features of the echo wave signals should be analyzed in accordance with the wideband signal requirements. Especially, as general signal

4

processing is done at the back end, a common wideband radar can always meet the condition of Eq. (3). Figs. 1 and 2, respectively, show oscillograms of the wideband signal and narrow-band signal.
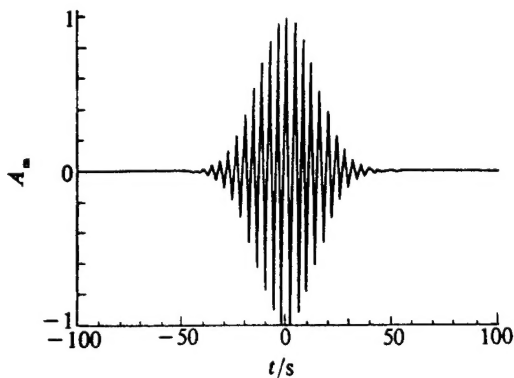


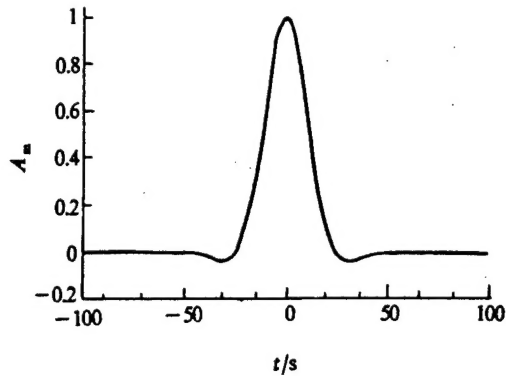Fig. 1. Narrow-band Signal            Fig. 2.  Wideband Signal


The wavelet transform serves as a significant mathematical tool for the multiple resolution analysis of abruptly changing signals and non-smooth signals. With its unique features, the wavelet transform, while applied to signal processing, shows the following advantages:

a. Since the wavelet transform is a linear transform, the features of the transformed signal are not distorted. It is particularly good for the non-smooth transient signals in that it can clearly single out the abruptly changing signals.

b. The wavelet transform can perform local analysis of signals in both the time domain and the frequency domain. In this way, a local detector is likely to arrive at a better result with lighter computational load.

c. The wavelet transform is mainly designed for use in processing the wideband (large product of time width by bandwidth) signals,

5

because the latter requires a high resolution rate in the time domain, while STFT is superior only when applied to narrow-band signal processing.

d. The wavelet transform, when used for analysis of local fineness, can accomplish a finer analysis of a short-time (finite data) signal. In other words, it is advantageous in short-time data analysis.

As the wavelet function is tightly supported while the Fourier base is not, the wavelet transform of normal white noise is still normal white noise, while the Fourier transform of normal white noise is not, which appears to be a very important feature of the signal analysis with the wavelet transform.

## 1.1 Wavelet Transform and Ambiguity Function

### 1.1.1 Definition of Narrow-band Signal Ambiguity Function

$$\chi(\tau, \zeta) = \int_{-\infty}^{\infty} u(t)u^*(t+\tau)\exp(j2\pi\zeta t)dt \tag{4}$$

where u(t) is the complex modulated envelope that is outputted by a signal through a matching filter, which is virtually a reciprocity function of target signal u(t) and target signal $u^*(t+\tau)\exp(j2\pi\zeta t)$ $(u(t)$ after time delay $\tau$ and Doppler shift $\xi$). It is to be noted that in the case of a narrow-band signal, WVD is in a two-dimensional Fourier transform relation with the ambiguity function $\chi(\tau,\zeta)$ [1], i.e.

$$P_{wv}(t, \omega) = \iint_{-\infty}^{\infty} \chi(\tau, \zeta)\exp[j(\zeta t - \omega\tau)]d\tau d\zeta \tag{5}$$

### 1.1.2 Definition of Wideband Signal Ambiguity Function

Based on the above analysis, for a wideband signal, the expression of the u(t) echo wave signal form through the Doppler shift is
u(ρ(t-τ,)), where ρ is the time extension factor. Thus, the ambiguity function at that time can be defined as [17-19]:

$$\chi(\tau, \rho)=\int_{-\infty}^{\infty} u(t)u^*(\rho(t-\tau))dt \qquad (6)$$

$$\rho=(c\pm v)/(c\mp v) \qquad (7)$$

where c and v, respectively, are the speed of light and the target velocity relative to the radar; ρ >1 in forward motion and ρ<1 in relative motion.

It can be seen from Eq. (6) that the self-ambiguity function of the wideband signal has a form similar to the wavelet transform. The difference is that the ambiguity function requires the wavelet function to be the transformed signal itself, while in the wavelet transform, the transform (wavelet) function is different from the transformed function. In this sense, the wavelet transform can also be referred to as the wideband reciprocity function. Therefore, the wavelet transform theory can be used to analyze the self-ambiguity function and the reciprocity function of the signal and further provide support for the radar waveform design.

## 1.2 Application of Wavelet Transform to Signal Detection and Parameter Estimation

### 1.2.1 Signal Detection Based on Wavelets

Suppose the received signal is expressed in the following two forms [20]:

$$H_1: x(t)=s(t)+c(t)+n(t) ; \qquad H_0: x(t)=c(t)+n(t) \qquad (8)$$

where x(t), s(t), c(t) and n(t) respectively are the received echo wave signal, the signal to be detected, the clutter and the

system thermal noise in L-dimensional space.

Suppose the vector c(t) can be simulated as a zero mean value, the covariance matrix is Gaussian distribution process of $R_c$, while s(t), c(t) and n(t) are assumed to be mutually independent, then the corresponding plausibility ratio will be [20]:

$$L(x) = x^T(t)(R_d^{-1} - (R_d + R_s)^{-1})x^*(t) \tag{9}$$

where $R_d \triangleq R_c + I$; T is a transposed vector; * is complex conjugate. When the clutter spectrum is very wide, i.e. $R_d = \sigma_d^2 I$, the corresponding plausibility ratio will become:

$$L(x) = x^T(t)s^*_{H_1}(t)_{H_1} > D \tag{10}$$

where $s^*_{H_1}(t)_{H_1} = E\{s(t)/x(t), H_1\}$, $E\{\cdot\}$ is the selected statistic mean value, D is the detection threshold.

This is the conventional signal detection method. In fact, it is possible to transform these data, with the wavelet transform method, into the two-dimensional time-frequency phase space domain to be analyzed. By expanding Eq. (8) in accordance with the discrete wavelet base, the following can be derived:

$$H_1 : x_{j,k}(t) = s_{j,k}(t) + c_{j,k}(t) + n_{j,k}(t) \ ; \quad H_0 : x_{j,k}(t) = c_{j,k}(t) + n_{j,k}(t) \tag{11}$$

Due to the property of the wavelet, Eq. (10) can be changed to [20]:

$$L(x) = \frac{1}{c_W} 2^{-j} \sum_j \sum_k \langle x(t), W_x(j, k) \rangle \cdot \langle s_{H_1}(t), W_x(j, k) \rangle \tag{12}$$

where $c_W$ is the constant term which constrains the wavelet from satisfying the permitted condition.

Then, how do we select factors j and k?  Assume the range of the wavelet transform used is -J≤j≤J, -K≤k≤K, then a block method can be used in response to the features of the wavelet transform.

## 1.2.2 Parameter Estimation Based on Wavelets

First, a definition should be given to the time-scale energy distribution function

$$\eta(j,k)=|W_x(j,k)|^2 \tag{13}$$

then, the "average scale factor " at each moment is

$$\bar{j}=\sum_k j\eta(j,k)/\sum_k \eta(j,k) \tag{14}$$

the "average translated factor" of each scale is

$$\bar{k}=\sum_j k\eta(j,k)/\sum_j \eta(j,k) \tag{15}$$

As a result, Eqs. (14) and (15) can be used to estimate the Doppler frequency and Doppler rate of radar signals like FMCW.

## 1.3 Wavelet Transform and Target Identification

When a radar is identifying a target, it has to single out those echo wave data of the identified target that can reflect the essential features of the target, namely: (1) the monotypical target pattern stays relatively stable and (2) different types of target patterns stay remarkably different.  Only these features are favorable for identification.  The data domain on which a radar is based in target identification involves the time (air) domain, the frequency domain, the polarization domain, etc.  Here the frequency domain basically refers to the Fourier domain, which can eliminate the translational component of the target relative to the radar (reflected on the phase of the Fourier domain) after the data are modelled and have realized the translational invariance.  However, the Fourier transform is a

9

frequency spectrum analysis that takes the time domain data as a whole without any response to the local information of the target echo wave data, which is due to the non-local analysis character of the Fourier transform. Yet the local information of the target is virtually an extremely important feature in target identification because it reflects the essential difference among different targets. It is thereby necessary to extract this local characteristic information of different targets by using the wavelet transform as a "mathematical microscope" so as to improve the identification performance of the radar.

## 2. Prospects of Wavelet Transform and Signal Processing

The wavelet transform has wide application prospects in high-resolution and ultra-wideband radars (impact radar, noise radar and pulse compression radar) because the signal echo waves from these radars possess high-resolution local features, and can reflect the fine structure of the target or generate the one-dimensional, two-dimensional and three-dimensional images of the target. Once the wavelet transform theory is combined with the ultra-wideband radar technology, it will eventually transform the difficult situations in future intelligent target identification and detection and bring new vitality to radar technology.

Nevertheless, the wavelet transform theory is by no means limited to radar signal processing alone. It is also a significant mathematical tool in nonlinear signal processing including neural net, subform, etc. Such an organic combination will formulate the core of modern nonlinear intelligent signal processing. In short, the wavelet transform--the mathematical "microscope" technology--should include at least the following areas of signal processing [21-25]:
(1) Anomalous signal detection; (2) speech signal processing; (3) seismic prospecting signal processing; (4) image processing; (5) data compression and subcoding; (6) multiple subdimensional

10

features and scale analysis of subform and random signals; (7) self-adaptive filtering and array signal processing; (8) clutter analysis and radar low-elevation tracking; (9) system identification and spectrum estimation and (10) model separation and group velocity determination.

In addition, in the research on the wavelet transform theory, even more interesting is how to select the basic wavelet function with some particular features so that the wavelet transform can describe the function features to be transformed more conveniently. The key problem in the wavelet transform theory is how to construct such a wavelet function that can meet the local time-frequency features in solving practical problems. In this case, the difficulty is how to formulate a wavelet function that is interfacing with the object, which appears to be an urgent knotty problem that mathematicians and engineering experts are confronting.

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